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Coding Fun: Rearranging All The Numbers

Coding Fun: Rearranging All The Numbers

In issue 13 there were 8 problems involving sieves on the natural numbers; that is, schemes for extracting various sequences, similar to the Sieve of Eratosthenes for extracting the primes.

One of them was solved by Sanford Greenfarb and by David Ferguson in issue 19. That problem was "rediscovered" and appeared again as Problem 170 in issue 49, with general and specific solutions appearing in issue 51.

The following set of problems involve various ways of rearranging all the natural numbers. The cover diagram is for THROWBACK. For the unending stream of integers starting with 3, each number at the head of the stream (the leader) is to be moved back in the stream the number of places indicated by its value, as shown by the arrows. Eventually, every number will appear at the head of the list. For example, when 10 first appears, there will be this ordering:

10, 5, 3, 4, 7, 11, 6, 8, 12, 9, 13, 14, 15,...

and 14 numbers will have been moved back.

For THROWBACK, then, we have three sub-problems:

1. In order to continue the process until the number 100 first appears as the leader, how many numbers must be considered; that is, how much storage must be allocated for the solution?

2. What will be the ordering when the number 100 first appears as the leader?

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3. Extend the following table, which shows how many numbers have been moved when the number K first appears as the leader:

K	moves
4	1
5	2
6	3
7	5
8	7
9	10
10	14
11	19
12	26

(A casual investigation of this function suggests that when K is 100, the number of moves will be around 10^{12})

PROBLEM 193

Each of the REARRANGING problems forms an excellent exercise in computer coding. If carried to modest limits, each problem fits any computer and can be coded in any language.

In MULTIPLE THROWBACK, take the natural numbers starting with 3. Move the leading number back by its own number of places, as in THROWBACK. At the same time, move every multiple of the leading number back by the same number of places. This is to be done in the order of the multiples; that is, if the leading number is 3, then the numbers 3, 6, 9, 12, 15,... are all to be moved back three places in that order. Print the new leading number and delete it from the stream. Produce the first 500 leading numbers, a list that begins:

4, 3, 8, 5, 9, 7, 6, 17, 14, 11, 15, 18,...

After the first stage, the number 4 will be printed, and there will remain:

5, 3, 7, 8, 6, 10, 11, 9, 13, 14, 12, 16,...

After the second stage, 4 and 3 will be printed, and there will remain:

7, 8, 6, 5, 11, 9, 13, 14, 10, 12, 16, 17,...

PROBLEM 194

PROBLEM 195

In REVERSE, take all the natural numbers and reverse each consecutive pair, to produce this sequence:

2, 1, 4, 3, 6, 5, 8, 7, 10, 9, 12, 11, 14, 13,...

Print the first pair and delete them from the stream. For all those remaining, reverse them by threes, to produce:

6, 3, 4, 7, 8, 5, 12, 9, 10, 13, 14, 11,...

Print the first three and delete them from the stream. Of the numbers remaining, reverse them by fours, to produce:

12, 5, 8, 7, 14, 13, 10, 9, 16,...

Print the first four and delete them from the stream. Then continue by reversing by fives, sixes, and so on. The list that will be printed begins:

2, 1, 6, 3, 4, 12, 5, 8, 7, 16, 9, 10, 13, 14,...

Calculate and print the first 1000 terms of the sequence.

PROBLEM 196

In KNOCKOUT, take all the natural numbers, starting with 3. At each stage, let the leading number be K. Extract the number that is K numbers away from the leader, print it, and move the leader to that position. The first stages are as follows:

3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,...

Print the 6 and replace it by the 3:

4, 5, 3, 7, 8, 9, 10, 11, 12, 13, 14,...

Print the 8 and replace it by the 4:

5, 3, 7, 4, 9, 10, 11, 12, 13, 14, 15,...

Print the 10, and replace it by the 5:

3, 7, 4, 9, 5, 11, 12, 13, 14, 15, 16,...

The printed output begins:

6, 8, 10, 9, 14, 12, 4,...

Most, but not all, of the natural numbers will be printed. Produce the first 500 numbers of the output stream.

PROBLEM 197

In LEADER REVERSE, take all the natural numbers starting with 3. The value of the leader dictates the number of numbers to be reversed. Reverse them, and extract from the stream the new leader. The first few stages are:

5, 4, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14,...

7, 6, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15,...

10, 9, 8, 4, 3, 6, 11, 12, 13, 14, 15, 16,...

14, 13, 12, 11, 6, 3, 4, 8, 9, 15, 16, 17,...

19, 18, 17, 16, 15, 9, 8, 4, 3, 6, 11, 12, 13,...

25, 24, 23, 22, 21, 20, 13, 12, 11, 6, 3, 4, 8, 9,...

and the sequence 5, 7, 10, 14, 19, 25,... is to be extended for 500 terms.

☐

WIND CHILL

Exercise. According to the Environmental Data Service of the U.S. Department of Commerce, the equivalent temperature (i.e., the "Wind Chill" factor) is given by:

$$T_e = 33 - (.4743 + .4538\sqrt{v} - .04538v)(33 - T)$$

where 33 is the neutral skin temperature in degrees Celsius, v is the wind speed in meters per second, and T is the air temperature in degrees Celsius.

Construct a table of wind chill equivalent temperatures for air temperatures from -45 degrees F to +45 degrees F in steps of 5 degrees F, and wind speeds from 5 mph to 45 mph in steps of 5 mph.

Wind speed in meters/second is obtained from wind speed in mph by multiplying by .44704089.

Some test values are these:

Wind speed mph	Air temp F	Wind Chill equiv. temp
5	45	43
15	25	2
25	10	-29
30	-10	-64
45	-45	-125

PROBLEM 198

OWARE

Game playing with computers can be categorized this way:

A. Recreational games, ranging from "Guess the number" to "Lunar Lander" and "Star Trek."

B. Determinate games, including most forms of Nim and Tick-Tack-Toe. There is either an algorithm for optimum play, or all possible positions of the game could be stored (and the game played by table lookup), or a winning strategy could be obtained by exhausting all possible moves.

C. Games with a hidden or random element (e.g., Blackjack, roulette).

D. Open-board games of strategy, epitomized (indeed, apotheosized) by chess and checkers.

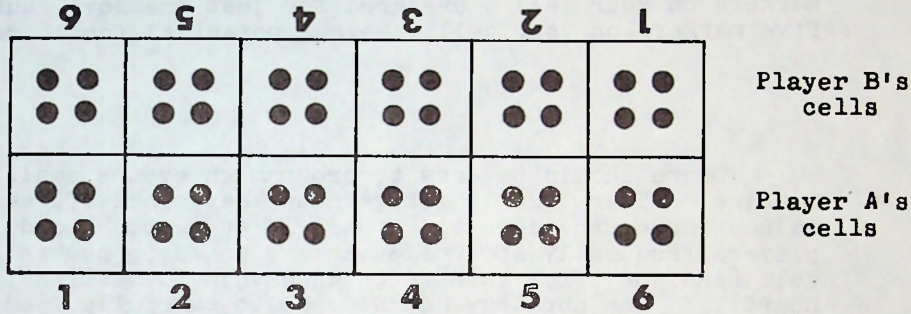
It is class D that offers opportunity for modest research with computers. It is postulated that such games can not be tackled by exhaustive methods--the number of possible moves grows exponentially, to the point where time is the limiting factor. However, it can be reasoned that these games do have a strategy for winning, if only for the fact that some players consistently win. It may be that no element of such a strategy is known (as for the game of Pasta, described in issue number 12). It is even conceivable that there is <u>not</u> a strategy of play; that winners simply are careful and avoid give-away moves, thereby winning by wearing out their opponents.

Chess and checkers represent extremes. Chess, for example, requires considerable programming just for its own mechanics; that is, for the mechanisms to make correct moves by each of the six kinds of pieces. Plus the fetish among chess players to have moves reported back in chess notation, rather than simply numbering the squares of the board. Both chess and checkers are complex games, better left to experts. Pasta, it is conjectured, is about half way between chess and checkers in its complexity.

But that leaves a lot of games to be explored in depth by computer programs. The game of Fives was suggested (issue 47, page 17).

Another interesting game that could be tackled is Oware. The game was first described in English journals in an article by W. W. Sawyer in Scripta Mathematica, June, 1949.

The playing board for Oware has twelve cells, six controlled by each player. The initial setup is shown here, with each cell containing four markers.



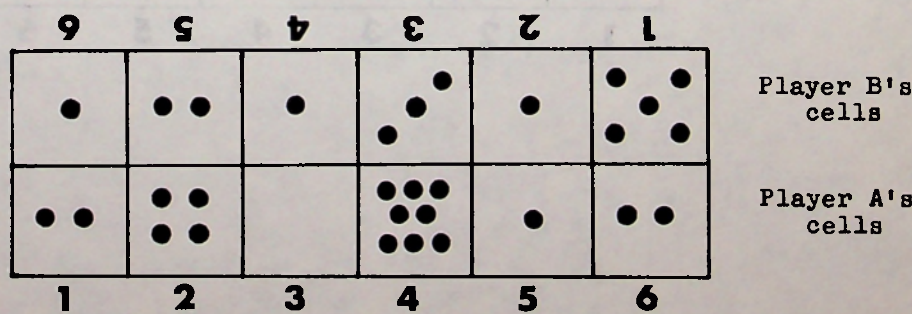
When it is a player's turn, he selects one of the six cells he controls, picks up all the markers in that cell (this number cannot be zero) and distributes them one to a cell, starting with the cell to the right of the one he chose and continuing counterclockwise as far as they go (but not returning any markers to the cell from which they came).

If the cell in which the last marker lands is

(A) One of his opponent's six cells, and

(B) If, after landing, there are two or three markers there, they are removed from play. Further, if the cell immediately preceding fulfills both conditions, its markers are removed, and so on back until both conditions are not met.

For example, for this position:



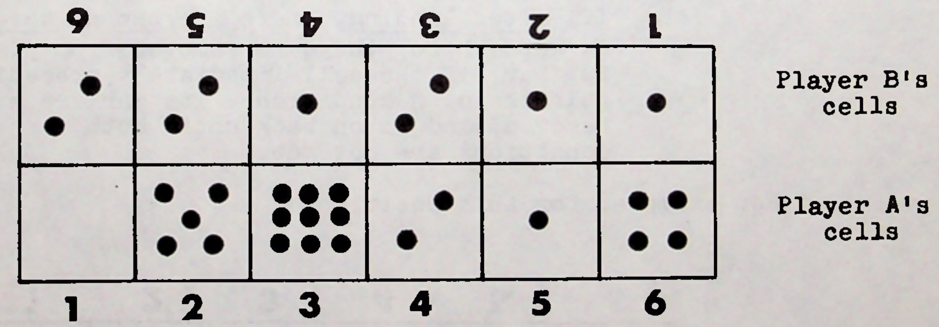
if A plays his 4, all markers will be removed from B's cells 6, 5, and 4.

The object of the game is to keep moving. The first player unable to move loses.

Thus, it is moves that count, and not markers. Five markers on your cell 6 are good for just one move, but five markers on your cell 1 have a potential for 16 moves.

Oware should be easy to program on even a small machine. There are 12 integers to keep track of, whose values range from zero to perhaps 15 at most. Good players frequently stockpile markers on their number 2 cell (and the trick then is to know when to bust that hoard). The strategy seems to be to carefully feed markers to your opponent (most of which soon return to you) until the proper time comes to starve him so that he runs out of markers, and hence runs out of moves.

Most games played by careful players wind up with the outcome hinging on just one move; that is, if A loses but would have had just one more move, A would have won. On the other hand, a game can end with a grand-slam play, such as A's 3 in this situation:



By All Means

The arithmetic mean of two numbers, X and Y, is their average:

$$AM = \frac{X + Y}{2}$$

The geometric mean of two numbers, X and Y, is the square root of their product:

$$GM = \sqrt{XY}$$

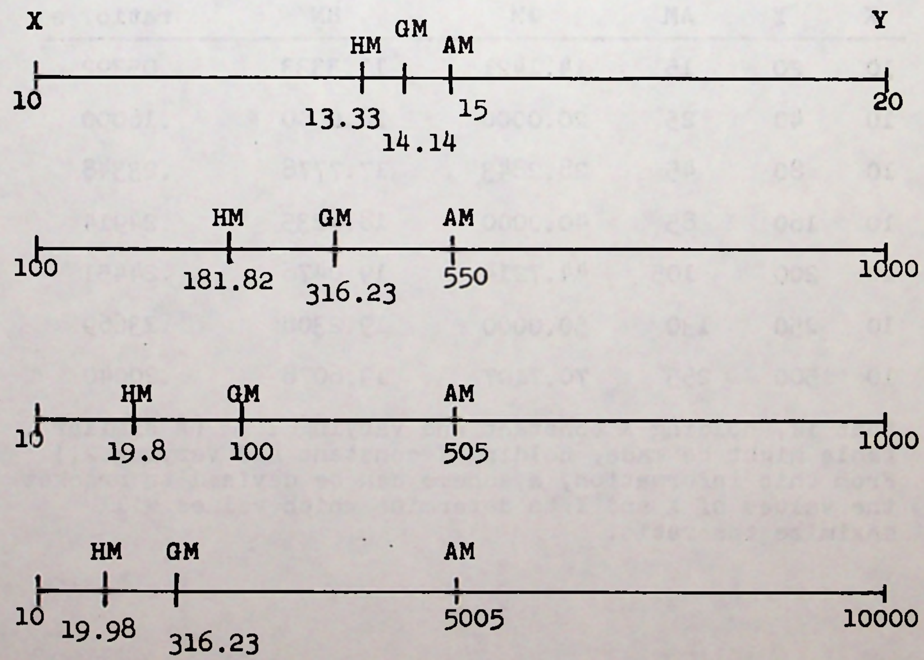
The harmonic mean of two numbers, X and Y, is such that 1/X, 1/HM, and 1/Y are in arithmetic progression, which implies that

$$HM = \frac{2XY}{X+Y}$$

The three means are equal when X = Y. Otherwise, for positive numbers, they generally have the relation

$$HM < GM < AM$$

Thus, we can have situations like the following:



Consider the ratio:

$$\frac{GM - HM}{AM} = s$$

The ratio s is zero when either X or Y is zero, or when X = Y. What values of X and Y will make this ratio the largest?

This problem can be attacked analytically, but it is suggested as a computing problem--an exercise in designing a computer solution to a problem. We suggest an attack like the following:

X	Y	AM	GM	HM	ratio, s
10	20	15	14.1421	13.3333	.05392
10	40	25	20.0000	16.0000	.16000
10	80	45	28.2843	17.7778	.23348
10	160	85	40.0000	18.8235	.24914
10	200	105	44.7214	19.0476	.24451
10	250	130	50.0000	19.2308	.23669
10	500	255	70.7107	19.6078	.20040

that is, holding X constant and varying Y. (A similar table might be made, holding Y constant and varying X.) From this information, a scheme can be devised to bracket the values of X and Y to determine which values will maximize the ratio.

The various means can be manipulated in another interesting way.

For two positive numbers, X and Y, calculate the geometric mean and the harmonic mean. Then let the GM be the new X and the HM be the new Y, and calculate those means again. As this procedure repeats, the two means converge rapidly, and the resulting single number is a function of the ratio Y/X. For example:

X	Y	Converges to:
10	200	27.910323
100	2000	279.10323
4321	86410	12059.73325
57	1140	159.08884
570	11400	1590.8884

As a rough approximation, the convergence value, C, is related to the ratio of Y/X, R, as follows:

$$C = \left[1.4658 \log R + .88371\right] X$$

In any event, it is relatively easy to calculate a table:

Y/X	Converge to
10	2.3527158
100	3.8143642
1000	5.2801572
10000	6.746027
100000	8.211898

But the inverse table is not so easy to calculate:

Y/X	Converge to
	2
	3
	4
643.99	5
	6
	7
	8
	9
	10

We have here another fine example of the utility of the bracketing process (see issues 35, 39, and 44 for discussions of this technique).

PROBLEM 201

And here's a third problem involving the three means. When the ratio of Y to X is small:

X	Y
10	50

HM = 16.6667
GM = 22.3607
AM = 30.0000
69.0274

the sum of the three means exceeds Y. When the ratio is large, on the other hand:

X	Y
10	200

HM = 19.0476
GM = 44.7214
AM = 105.0000
168.7690

the sum is less than Y.

For some value of Y/X, the three means sum to exactly Y. What is the value of that ratio?

PROBLEM 202

Given eight words of storage, numbered from 1 to 8. The contents of the eight words form a permutation of eight things. This permutation can be turned into a different permutation by interchanging two adjacent words; the pair thus exchanged is identified by giving the position of the first of the pair. Thus, the transformation from

1 2 3 4 5 6 7 8
to
1 2 3 5 4 6 7 8

is identified by the number 4.

The initial arrangement (12345678) can be turned into any other arrangement by, at most, 28 moves. The chart shows the 28 steps going from ascending to descending order.

A subroutine is wanted for which the calling sequence specifies the desired arrangement and the subroutine is to furnish the pair identifications of the steps that will accomplish the transformation. The starting arrangement is always 12345678.

For example, if the subroutine is called with 38152467, the subroutine should produce a string like 217625432354. The output string should be the shortest possible number of steps.

The subroutine may be written in any programming language. Its logic should be explained, via a flowchart or equivalent device, and a thorough test procedure should be included.

input arrangement

1 2 3 4 5 6 7 8

2 1 3 4 5 6 7 8	1
2 3 1 4 5 6 7 8	2
2 3 4 1 5 6 7 8	3
2 3 4 5 1 6 7 8	4
2 3 4 5 6 1 7 8	5
2 3 4 5 6 7 1 8	6
2 3 4 5 6 7 8 1	7
3 2 4 5 6 7 8 1	1
3 4 2 5 6 7 8 1	2
3 4 5 2 6 7 8 1	3
3 4 5 6 2 7 8 1	4
3 4 5 6 7 2 8 1	5
3 4 5 6 7 8 2 1	6
4 3 5 6 7 8 2 1	1
4 5 3 6 7 8 2 1	2
4 5 6 3 7 8 2 1	3
4 5 6 7 3 8 2 1	4
4 5 6 7 8 3 2 1	5
5 4 6 7 8 3 2 1	1
5 6 4 7 8 3 2 1	2
5 6 7 4 8 3 2 1	3
5 6 7 8 4 3 2 1	4
6 5 7 8 4 3 2 1	1
6 7 5 8 4 3 2 1	2
6 7 8 5 4 3 2 1	3
7 6 8 5 4 3 2 1	1
7 8 6 5 4 3 2 1	2
8 7 6 5 4 3 2 1	1

identification number of the pair that were interchanged to produce this arrangement

PERMUTATATIONS

Exploring Random Behavior -- 2

So you think you understand how random processes will behave?

We are presenting a series of problems (starting in issue 54) designed to illustrate that people's intuitive grasp of random behavior may be misleading.

It is assumed that we have access to a pseudo-random number generator that will furnish a very long stream of numbers that are uniformly distributed in the range from zero to one. An adequate generator might be the one of Dr. Mordecai Schwartz:

$$\text{new number} = \text{fractional part}\left[\text{old number} + \pi\right]^5$$

which was analyzed in issue 43. Dr. Schwartz's generator passes most of the standard tests of randomness (these tests were listed in issue 33), but is weak on some of them (e.g., the leading digits do not pass the serial test). For most purposes, the generator will serve, and it has the advantage of being readily programmed on pocket calculators.

Given such a generator, certain tasks involving random behavior can be handled in a straightforward manner, and the results conform to our intuitive notions. Call the output of the generator X:

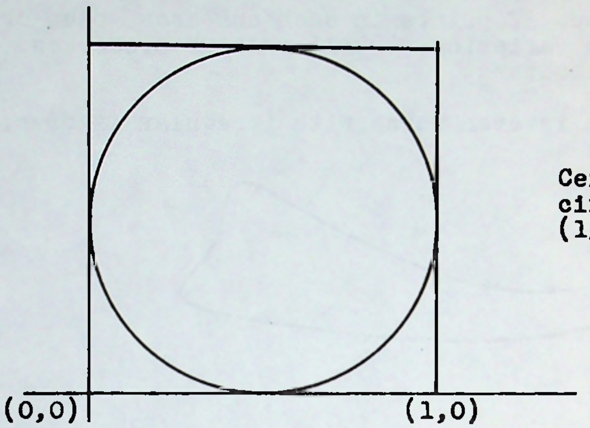
1. Pick a point at random on a line. If the line is of length L, the point is located by LX.

2. Pick a point at random in a square. If the square has side L, then the x-coordinate of the point is X_1L; the y-coordinate is X_2L.

3. Select one of K possible conditions at random. Change X into an integer (e.g., multiply X by 100000). Divide by K and add 1 to the remainder. This yields an integer in the range from 1 to K at random. For K = 6, for example, this process simulates the action of a cubical die.

We thus have the necessary tool to make a random selection, or to locate a random point in the simplest of geometric situations.

Suppose, now, we need to locate a point at random in a circle:



Center of the circle at (1/2, 1/2)

If the circle has unit diameter, located as shown, we might try two different approaches:

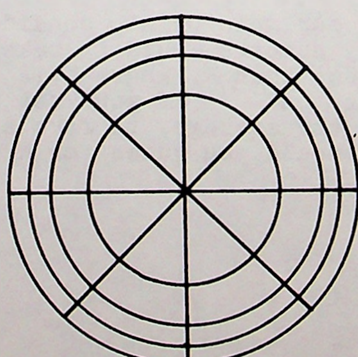
1. Select a point at random in the unit square, as described above. Then if

$$(x - .5)^2 + (y - .5)^2 \le .25$$

then the point (x,y) is in or on the circle. If not, reject the point and select another.

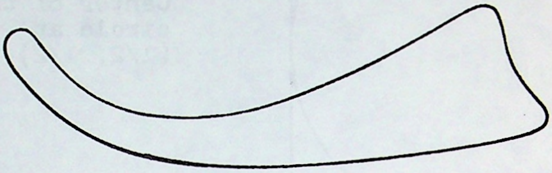
2. Select an azimuth and range at random to locate the point (x,y). If the direction straight up is taken as zero, a random azimuth in radians is $2\pi X_1$. The value $X_2/2$ will give a random range from the center of the circle.

Now, are these equivalent processes? If 1024 points are selected in each of two circles by these two processes, could any difference be detected in the two distributions? For example, if each circle were divided into 32 equal-area parts:



we would expect about 32 points in each sub-area, plus or minus normal random variation. Will the two processes give us the same results?

The situation is even worse with irregular figures, such as:



and the glib phrase "select a point at random on the perimeter" describes an action that may not be easy to perform, much less justify.

We have postulated a generator that outputs numbers uniformly distributed in the range from zero to one. In theory, a large set of such numbers should have a mean of .5 and a standard deviation of .2888. For example, 1000 values taken from our reference generator (with a starting value of zero) show the following:

sum = 490.008

sum of squares = 321.018

mean = .49

standard deviation = .2846

To illustrate our main point, consider this simple situation: how many numbers would it take to form a continued product less than K, where K is .01, .001, .0001, .00001, and so on? If you knew that it takes 17 such numbers, on the average, to form a product less than .0000001, what would you guess for K = .00000001?

If random numbers are needed having a normal distribution with mean M and standard deviation S, then:

$$R = M + (X_1 + X_2 + \cdots + X_n - n/2)\left\{\frac{2S}{\sqrt{n/3}}\right\}$$

where the X's are the usual output. Taking n = 12, we have:

$$R = M + S(X_1 + X_2 + \cdots + X_{12} - 6)$$

formed from 12 successive draws from the uniformly distributed generator.

Three problems in random behavior have been presented previously; they have not been solved:

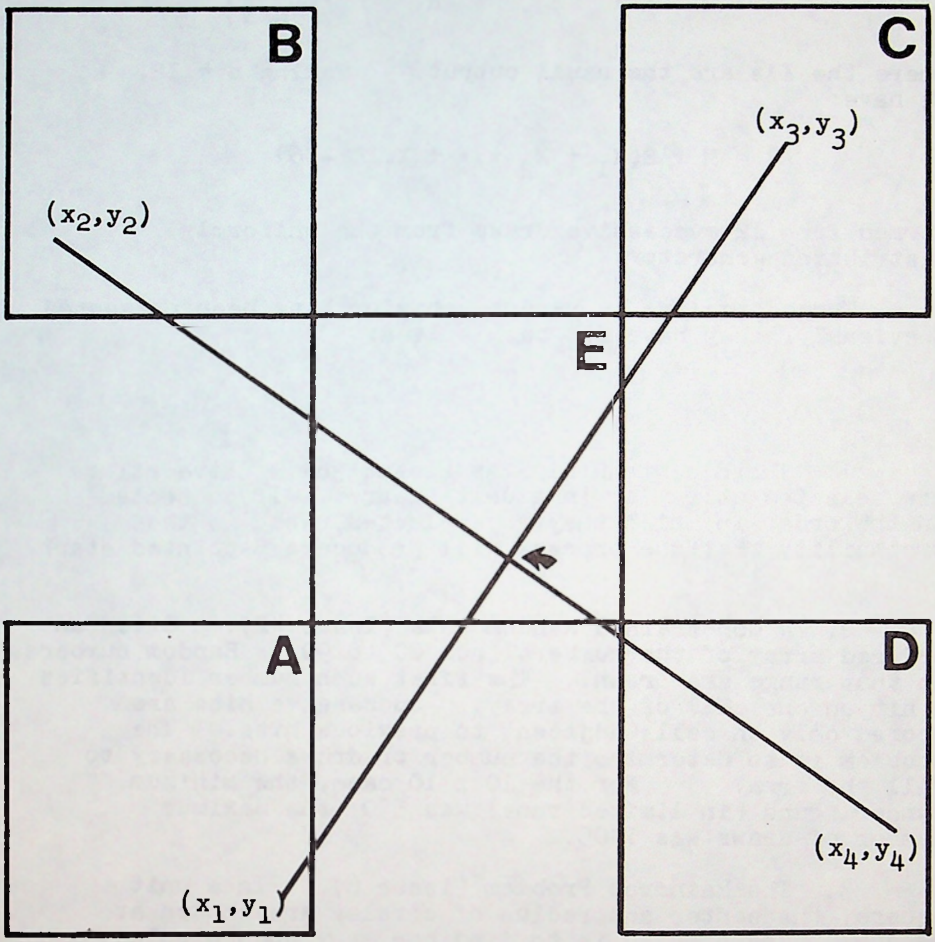
1. The Bicentennial Star (issue 36). Five points are selected at random in a unit square. If connected in the order in which they are selected, what is the probability that the process will produce a 5-pointed star?

2. A Constrained Random Walk (issue 42). Given an ordered array of the numbers from 00 to 99. Random numbers in that range are drawn. The first such number identifies a hit on one cell of the array. Successive hits are scored only on cells adjacent to previous hits. The problem is to determine the number of draws necessary to fill the array. For the 10 x 10 case, the minimum number found (in limited runs) was 550; the maximum number of draws was 1405.

3. The Raindrop Problem (issue 6). In a unit square, the center and radius of circles are chosen at random. The problem is to find how many such circles have to be taken to cover the square completely.

A new problem is illustrated in the accompanying diagram. Five unit squares are arranged as shown. Points are selected at random in squares A, B, C, and D, and are connected by pairs. What is the probability that the point of intersection of the two lines will fall in or on square E? Further, for those cases in which the intersection does fall in square E, will it determine a random point in the same sense as the choices made for squares A, B, C, and D?

PC55-18



Another excursion into random behavior.
How often will the point indicated by
the arrow fall outside square E?

PROBLEM 205

A Function Of Primes

Define K(p) to be the prime that is p primes beyond the prime p. Thus:

$$K(3) = 11$$
$$K(11) = 53$$
$$K(53) = 347$$
$$K(97) = 673$$
$$K(997) = 9419$$
$$K(8377) = 98101$$
$$K(5611997) = 104394557$$

The ratio of K(p) to p grows from 3.67 to 18.6 in this range.

Construct a table of K(p). Form a conjecture as to the growth of the ratio of K(p) to p for large values of p.

PROBLEM 206

It is interesting to conjecture about the size of K(p) for the largest known prime:

$$2^{19937} - 1$$

(given in decimal form--6002 digits--in issue 19).

"Impossible" is a dangerous word in computing, but it seems safe to say that K(p) for that prime will never be known.

The purpose of computing is insight, not numbers.

— Richard Hamming

The number of computations without purpose is out of sight.

— Gio Wiederhold

COMPOSITE TWINS

Calculate a list of composite odd integers that are not divisible by 3 or 5, beginning with the list shown here.

The list contains a pair of twins. What are the next four sets of twins on the list?

49
77
91
{119
121}
133
143
161
169
187
203
209
217
221

PROBLEM 207

MENTAL COMPUTING

Take a number, x, between zero and one. Considering x as an angle in radians, calculate its sine. Add a constant, K, to produce a new value of x, and repeat the process. In other words, we have the recursion:

$$x_{n+1} = \sin x_n + K$$

For the proper choice of K, this will converge for any non-zero starting value of x. Given that fact, find K--without using a machine, or even pencil and paper.

PROBLEM 208

SUMS OF FIVE

The following problem comes from James L. Boettler, Talladega College, Talladega, Alabama:

"Find a set of five positive distinct integers such that every pair of them sums to a perfect square.

"I have worked on this problem and did manage to get many sets of four integers, such as

2, 167, 674, 6722

18, 882, 2482, 4743

"There are many 4-integer solutions in which the smallest integer is of the form $2n^2$. But others have the smallest integer 16, 23, 63, 88, 92, 95..."

PROBLEM 209